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ORIGINAL ARTICLE

Multivariate process capability analysis applied to AISI 52100 hardened steel turning

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Abstract
Hard turning operations have been extensively investigated owing to their ability to reduce process cycle time, increase process flexibility, ensure high-dimensional accuracy, and enable machining without a cutting fluid. These processes are rather common for dealing with multiple quality characteristics. To evaluate the process ability and meet customer needs, multivariate statistical techniques are recommended for estimating the capability indices. Principal component analysis can be applied to reducing the problem dimension and estimate process capability indices. The aim of this study was to assess the capability of AISI 52100 hardened steel turning operations and achieve process specifications. Multivariate process capability indices were calculated to assess five roughness parameters of surface finishing. By using a weighted approach of principal component analysis, a new method is proposed for estimating the process capability indices. The results highlight not only the relevance of conducting a multivariate capability analysis in the case of actual machining but also how successfully the proposed method was performed.

Keywords Process capability index · Hard turning · Roughness · Principal component analysis

1 Introduction

Recently, extensive attention has been focused on the understanding of hard turning processes [1–4]. The traditional machining of hardened materials usually requires rough turning, heat treatment, and grinding processes. Nevertheless, hard turning can eliminate part of these processes. As a result, the process cycle time is reduced, while productivity is increased [4]. This operation has become an important manufacturing process within a wide range of industrial applications such as gears, shafts, bearings, cams, forged parts, molds, and dies [5]. In this process, material hardness is usually greater than 45 HRC [6]. The turning operation is performed with advanced tool materials, such as mixed ceramics (Al_2O_3 + TiC) and cubic boron nitride (CBN), which induce significant benefits such as short cutting time, process flexibility, adequate surface roughness, high material removal rate, dimensional accuracy, and machining without a cutting fluid [7]. Additionally, hard turning takes advantage of modern machine tool operation, which allows the manufacturing of products with a high complexity of geometries and shapes [8].

Some authors have applied process capability analysis in order to estimate the machining process ability and operate within specifications [9–13]. However, thus far, studies on process capability analysis applied to the hard turning process have been limited. Corporations and firms committed to continuous quality improvement have adopted statistical process control to predict the current and future states of a

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process. Several tools such as control charts and process capability indices have been used to statistically evaluate a process. The latter was applied to assess whether a process is able to meet process/product requirements and hence satisfy customer needs [14–16].

Currently, a massive amount of data has been generated through automated industrial processes. As the complexity of the problem increases, methods that are more comprehensive are required for dealing with such a challenging environment [17, 18]. As a result, novel research should take into account multivariate control charts, data mining tools, Markov Chains, advanced process capability analyses, etc. [18]. New multivariate process capability indices, based on principal component analysis, have been proposed since the first work by [19] and include papers [20–34].

Like most machining processes, hard turning deals with several correlated quality characteristics [7, 35]. To assess whether such a process is able to produce good parts, a multivariate approach must be conducted to estimate the process capability indices. This research aims to evaluate the capability of the AISI 52100 hardened steel turning process with regard to producing parts within the specifications for correlated roughness parameters. A new method, based on weighted principal component analysis, was applied to estimate process capability indices. A comparison study was performed against some multivariate indices from the literature to validate the proposed method. The results of the assessment of the hard turning process capability demonstrated the adequacy of the proposed method.

The remainder of this paper is structured as follows. Section 2 presents a way to evaluate the multivariate process capability based on principal component analysis. In Section 3, the proposed *WPC* method is highlighted. Section 4 describes the multivariate hard turning experiments. Section 5 describes how the multivariate methods are applied to evaluate whether the hard turning operation meets the specifications. Finally, Section 6 outlines the main findings of this study.

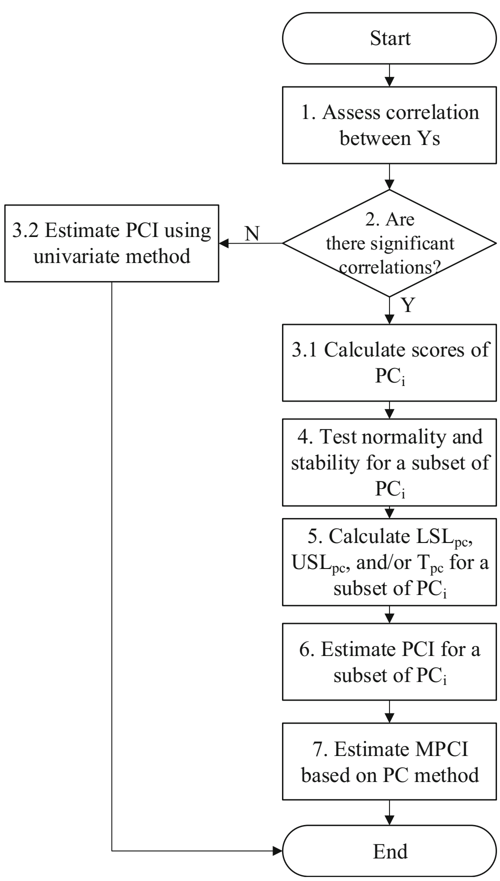


Fig. 1 Multivariate process capability indices based on *PC* methods

2 Multivariate process capability analysis

Let Y be a univariate quality characteristic, having a mean μ and variance σ^2. T, LSL, and USL are considered as target values, lower specification limits, and upper specification limits, respectively. Univariate process capability indices, based on normal distribution, can be estimated by a combined formulation, such as that in [14, 21, 36]:

$$Cp(u,v) = \frac{d - u|\mu - T|}{3\sqrt{\sigma^2 + v(\mu - T)^2}}. \tag{1}$$

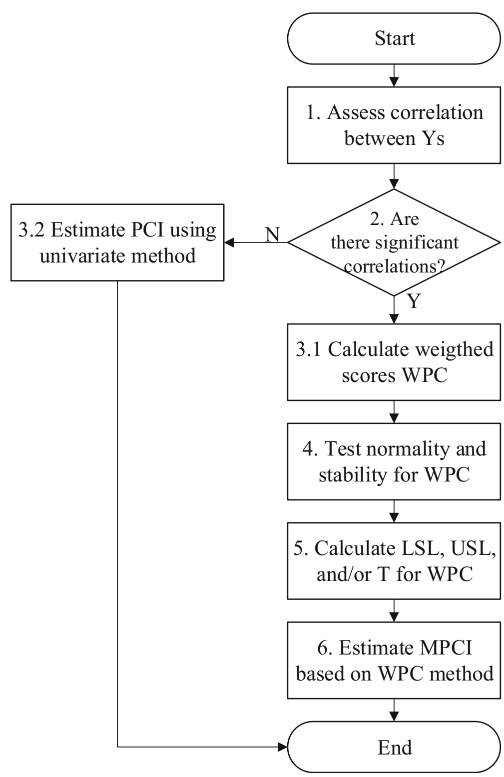


Fig. 2 Multivariate process capability indices based on proposed *WPC* method

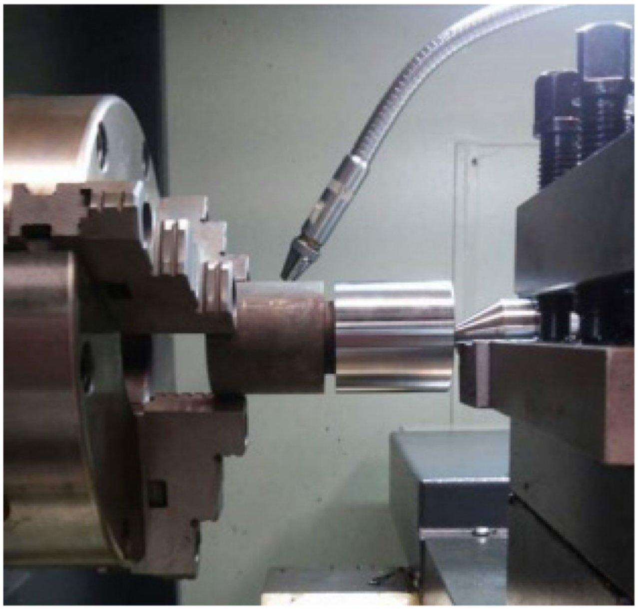


Fig. 3 AISI 52100 hardened steel turning operation

In this expression, $d = (USL - LSL) / 2$, $T = (USL + LSL) / 2$ and (u,v) are two non-negative parameters. By assuming a value of 0 and/or 1 for parameters u and v, Eq. (1) becomes:

$$Cp(0,0) = C_{\mathrm{p}} = (USL{-}LSL)/(6\sigma) \tag{2}$$

$$Cp(1,0) = C_{\mathrm{pk}} = (d{-}|\mu{-}T|)/(3\sigma) \tag{3}$$

$$Cp(0,1) = C_{\mathrm{pm}} = (USL{-}LSL)/\left[6\sqrt{\sigma^2 + (\mu{-}T)^2}\right] \tag{4}$$

$$Cp(1,1) = C_{\mathrm{pmk}} = (d{-}|\mu{-}T|)/\left[3\sqrt{\sigma^2 + (\mu{-}T)^2}\right]. \tag{5}$$

In multivariate context, by considering that $\mathbf{Y}' = (Y_1, Y_2, \ldots, Y_m)$ represents the vector of q quality characteristics with a vector of mean $\boldsymbol{\mu}$ and positive definite variance-covariance matrix $\boldsymbol{\Sigma}$. The vectors of the target value, lower specifications, and upper specifications are $\mathbf{T}' = (T_1, T_2, \ldots, T_q)$, $\mathbf{LSL}' = (LSL_1, LSL_2, \ldots, LSL_q)$, and $\mathbf{USL}' = (USL_1, USL_2, \ldots, USL_q)$, respectively. The scores of principal components are calculated by:

$$\mathbf{PC}_{\mathrm{i}} = \mathbf{e}_{\mathbf{i}}'\mathbf{Y} \tag{6}$$

where $\mathbf{e}_{\mathrm{i}}$ are the eigenvectors of each $\mathbf{PC}_{\mathrm{i}}$, and $\mathbf{Y}$ may take a standardized form if the correlation matrix is used to estimate the scores of principal components. Multivariate specification limits and the target value in terms of principal components and $\mathbf{PC}_{\mathrm{i}}$ are obtained by [21, 23, 25, 26, 28, 32, 34]:

$$\mathbf{LSL}_{\mathrm{PC_i}} = \mathbf{e}_{\mathbf{i}}'\mathbf{LSL} \tag{7}$$

$$\mathbf{USL}_{\mathrm{PC_i}} = \mathbf{e}_{\mathbf{i}}'\mathbf{USL} \tag{8}$$

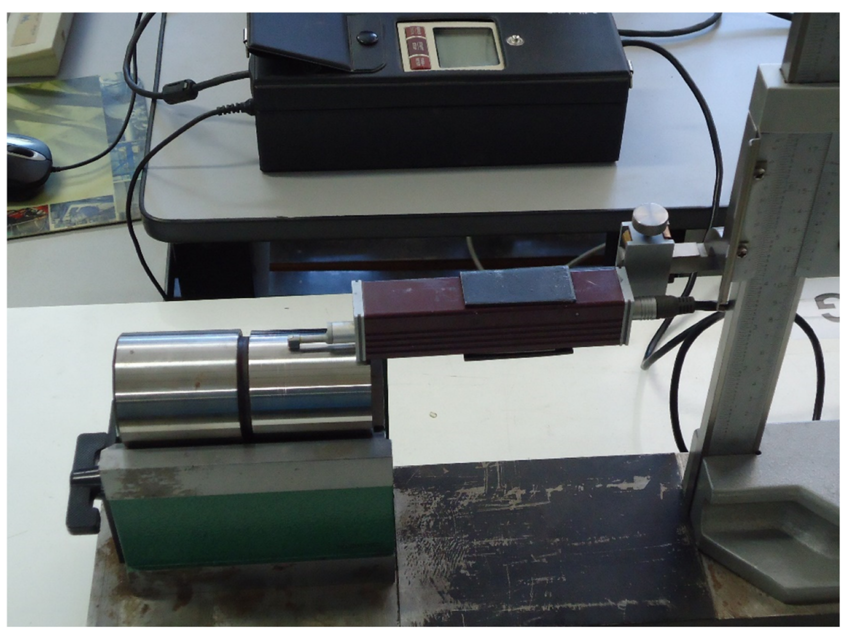
$$\mathbf{T}_{\mathrm{PC_i}} = \mathbf{e}_{\mathbf{i}}'\mathbf{T}. \tag{9}$$

Wang and Chen [19] proposed the evaluation of multivariate process capability by considering the subset υ ($\upsilon \le q$) of principal components. They defined MC_{p}, MC_{pk}, MC_{pm}, and MC_{pmk}, by using the univariate process capability indices for each principal component. A combined formulation of the multivariate process capability indices is defined by [19], as follows:

$$M_1Cp(u,v) = \left[\prod_{i=1}^{\upsilon} Cp_{\mathrm{PC_i}}(u,v)\right]^{1/\upsilon}. \tag{10}$$

In this expression, the process capability index $Cp_{\mathrm{PC_i}}(u,v)$ is estimated by Eq. (1); however, $d = (USL_{\mathrm{PCi}} -$

Fig. 4 Mitutoyo portable roughness checker model Surftest SJ-201P



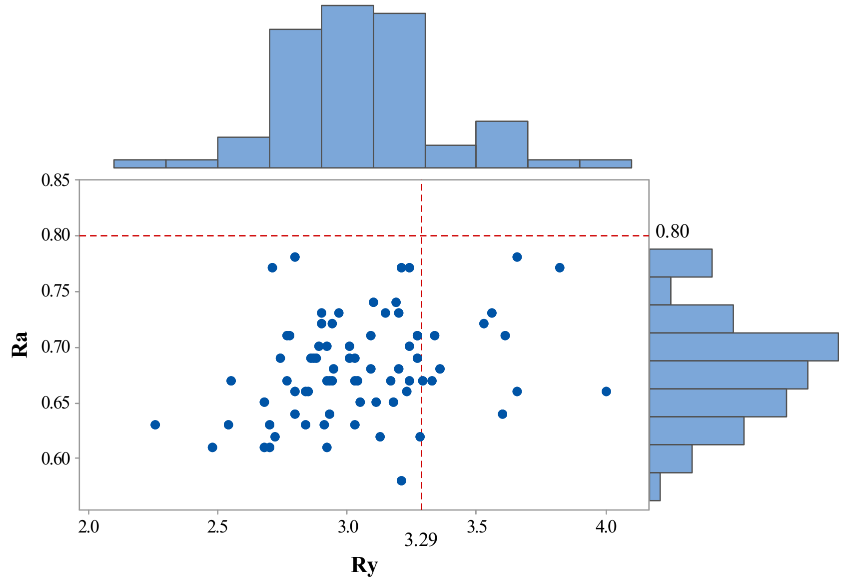


Fig. 5 Scatter plot and histograms for roughness parameters R_a and R_y

LSL_{PCi}) / 2, $T = (USL_{PCi} + LSL_{PCi})$ / 2, $\mu_{PC_i} = \mathbf{e}_i^{'}\boldsymbol{\mu}$, $\sigma_{PC_i} = \sqrt{\lambda_i}$, and λ_i are the eigenvalues of each PC_i, for $i = 1, 2, \ldots, \upsilon$.

The shortcoming of this capability index is that the principal components are equally weighted. As it is widely known, the first principal components are more relevant than the latter. To overcome this problem, [37] proposed the use of the weighted geometric mean. These weights are based on the eigenvalues λ_i of each principal component, as follows:

$$M_2C_p(u,v) = \left(\prod_{i=1}^{\upsilon} Cp_{PC_i}(u,v)^{\lambda_i}\right)^{1/\sum_{i=1}^{\upsilon}\lambda_i}. \tag{11}$$

In the same context, Perakis and Xekalaki [31] proposed the calculation of multivariate capability indices by using the weighted arithmetic mean. The weights were also based on the eigenvalues λ_i of each principal component, as follows:

$$M_3Cp(u,v) = \sum_{i=1}^{\upsilon}\psi_i Cp_{PC_i}(u,v) \tag{12}$$

where $\psi_i = \lambda_i / \sum_{j=1}^{\upsilon}\lambda_j$ is the explanation percentage of the ith principal component.

Additionally, Perakis and Xekalaki [31] highlighted some drawbacks of estimating the multivariate capability indices for processes with a one-sided specification. When

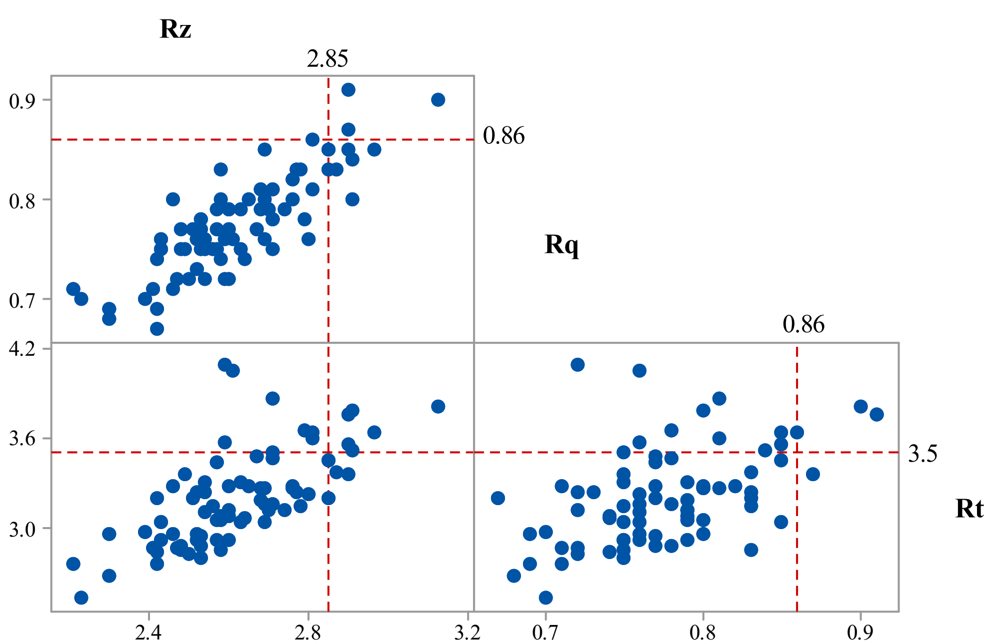


Fig. 6 Scatter plot for roughness parameters R_z, R_q, and R_t

estimating lower (upper) specification limits, some principal components may take the place of an upper (lower) specification limit, due to axes rotation. Therefore, the authors suggest taking the absolute $\left|Cp_{\mathrm{PC_i}}(u,v)\right|$ in Eqs. (9)–(11) to estimate the multivariate process capability indices.

3 Process capability based on weighted principal component analysis

The aforementioned multivariate process capability indices were estimated for each principal component. Subsequently, an agglutination strategy was performed in order to report the multivariate index [19, 31, 37]. This procedure is summarized in Fig. 1.

In this study, the weighting approach was applied to principal component scores. Then, the process capability indices were estimated. This proposal was developed based on Peruchi et al. [38]; however, the authors developed a new multivariate procedure in order to conduct a distinct statistical quality technique; namely, measurement system analysis.

Unlike previous studies, the proposed weighting approach for estimating multivariate capability indices is described in the procedure shown in Fig. 2. In step 1, we check whether there are significant correlations between quality characteristics by using:

$$Corr_{\mathrm{Y_iY_j}} = \frac{Covar_{\mathrm{Y_iY_j}}}{\sqrt{\mathrm{Var_{Y_i}Var_{Y_j}}}} \quad \forall i = 1,2,\ldots,q; \quad j = 1,2,\ldots,q \tag{13}$$

where $\mathrm{Var_{Y_i}}$ and $\mathrm{Var_{Y_j}}$ are the ith and jth variances, and $Covar_{\mathrm{Y_iY_j}}$ represents the covariance between Ys.

In Step 2, if significant correlations exist between Ys, we proceed to Step 3.1, which marks the beginning of the *WPC* method for estimating the multivariate process capability indices. On the other hand, if there are no correlations between Ys, we proceed to Step 3.2 in order to use the univariate approach by Eqs. (1)–(5).

In Step 3.1, by considering the matrix of standardized data (**Z**) and the matrices of weights (**W**) and eigenvectors (**e**), the weighted scores of the principal components are calculated by:

$$\mathbf{WPC} = \mathbf{W}'\left(\mathbf{e}'\mathbf{Z}\right) \tag{14}$$

Table 1 Normality test and univariate process capability estimates

Variable	Normality test	*USL*	$\hat{\mu}$	$\hat{\sigma}$	*Cp*(1,0)
R_a	0.480[a] (0.228[b])	0.80	0.681	0.045	0.879
R_y	0.687 (0.070)	3.29	3.048	0.313	0.258
R_z	0.349 (0.467)	2.85	2.621	0.176	0.435
R_q	0.400 (0.354)	0.86	0.775	0.050	0.564
R_t	0.728 (0.055)	3.50	3.211	0.319	0.302

[a] Anderson-Darling test for normality

[b] *p* value for normality test

where

$$\mathbf{W} = \begin{bmatrix} \lambda_1/\sum_{j=1}^{q}\lambda_\mathrm{j} \\ \lambda_2/\sum_{j=1}^{q}\lambda_\mathrm{j} \\ \vdots \\ \lambda_\mathrm{q}/\sum_{j=1}^{q}\lambda_\mathrm{j} \end{bmatrix} \quad \mathbf{e} = \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1\mathrm{q}} \\ e_{21} & e_{22} & \cdots & e_{2\mathrm{q}} \\ \vdots & \vdots & \ddots & \vdots \\ e_{\mathrm{q}1} & e_{\mathrm{q}2} & \cdots & e_{\mathrm{qq}} \end{bmatrix}$$

$$\mathbf{Z} = \begin{bmatrix} \left(\frac{Y_{11}-\overline{Y}_1}{\sqrt{s_{11}}}\right) & \left(\frac{Y_{12}-\overline{Y}_2}{\sqrt{s_{22}}}\right) & \cdots & \left(\frac{Y_{1\mathrm{q}}-\overline{Y}_\mathrm{q}}{\sqrt{s_{\mathrm{qq}}}}\right) \\ \left(\frac{Y_{21}-\overline{Y}_1}{\sqrt{s_{11}}}\right) & \left(\frac{Y_{22}-\overline{Y}_2}{\sqrt{s_{22}}}\right) & \cdots & \left(\frac{Y_{2\mathrm{q}}-\overline{Y}_\mathrm{q}}{\sqrt{s_{\mathrm{qq}}}}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\frac{Y_{\mathrm{n}1}-\overline{Y}_1}{\sqrt{s_{11}}}\right) & \left(\frac{Y_{\mathrm{n}2}-\overline{Y}_2}{\sqrt{s_{22}}}\right) & \cdots & \left(\frac{Y_{\mathrm{nq}}-\overline{Y}_\mathrm{q}}{\sqrt{s_{\mathrm{qq}}}}\right) \end{bmatrix}.$$

To test the normality and stability of the *WPC* vector, Anderson-Darling and control chart tools can be implemented in Step 4. More details on these procedures can be found in the study by Montgomery [15]. In terms of principal component weighted scores, the target values and specification limits are obtained in Step 5 as follows:

$$LSL_{\mathrm{wpc}} = \mathbf{W}'\mathbf{LSL}_{\mathrm{pc}} \tag{15}$$

$$USL_{\mathrm{wpc}} = \mathbf{W}'\mathbf{USL}_{\mathrm{pc}} \tag{16}$$

$$T_{\mathrm{wpc}} = \mathbf{W}'\mathbf{T}_{\mathrm{pc}}. \tag{17}$$

Table 2 Correlation analysis for roughness parameters

	R_a	R_y	R_z	R_q
R_y	0.314[a]			
	0.006[b]			
R_z	0.682	0.675		
	0.000	0.000		
R_q	0.938	0.462	0.834	
	0.000	0.000	0.000	
R_t	0.312	0.876	0.682	0.477
	0.006	0.000	0.000	0.000

[a] Pearson correlation

[b] *p* value

Table 3 Principal component analysis for roughness parameters

	PC_1	PC_2	PC_3	PC_4	PC_5
Eigenvalues	3.519	1.137	0.189	0.124	0.030
Proportions	70%	23%	4%	3%	1%
Cumulative	70%	93%	97%	99%	100%
Eigenvectors	PC_1	PC_2	PC_3	PC_4	PC_5
R_a	0.416	− 0.547	− 0.46	0.022	− 0.561
R_y	0.419	0.521	− 0.276	0.688	0.055
R_z	0.497	− 0.034	0.821	0.074	− 0.269
R_q	0.476	− 0.402	− 0.044	− 0.066	0.778
R_t	0.422	0.516	− 0.189	− 0.718	− 0.063

Finally, the multivariate process capability indices based on weighted principal component analysis can be estimated in Step 6, as follows:

$$M_{\mathrm{wpc}}Cp(u,v) = \frac{d-u|\mu-T|}{3\sqrt{\sigma^2+v(\mu-T)^2}}. \tag{18}$$

In this expression, (u,v) are two non-negative parameters; namely, $d=(USL_{\mathrm{wpc}}-LSL_{\mathrm{wpc}})/2$ and $T=(USL_{\mathrm{wpc}}+LSL_{\mathrm{wpc}})/2$; $\mu_{\mathrm{wpc}}=\mathbf{W}'\left(\mathbf{e}_{\mathbf{i}}'\boldsymbol{\mu}\right)$, $\sigma_{\mathrm{wpc}}=\sqrt{\sum_{i=1}^{q}W_i\lambda_i}$, and λ_i are the eigenvalues of each PC_i for $i = 1, 2, \ldots, \upsilon$.

4 Hardened steel turning application

The multivariate machining process evaluated in this research was the AISI 52100 hardened steel turning in Fig. 3. The workpieces (1.03% C; 0.23% Si; 0.35% Mn; 1.40% Cr; 0.04% Mo; 0.11% Ni; 0.001% S; 0.01%) with dimensions of Ø 49 mm × 50 mm were machined with the Nardini CNC lathe, with a maximum rotational speed of 4000 rpm and cutting power of 5.5 kW. The workpieces were quenched and tempered such that their hardness was between 49 and 52 HRC, up to a depth of 3 mm below the surface. The controlled machining parameters were the cutting speed $S = 220\ \mathrm{m\,min^{-1}}$, feed rate $F = 0.30\ \mathrm{mm\ rev^{-1}}$, and cut depth $D = 0.225$ mm [7]. Wiper used mixed ceramic ($Al_2O_3 + TiC$) inserts (ISO code CNGA 120408 S01525WH) and inserts coated with a very thin layer of titanium nitride (Sandvik-Coromant GC 6050). The tool holder presented a negative geometry with an ISO code DCLNL 1616H12 and entering angle $\chi\ r = 95°$.

In this multivariate process capability analysis, five roughness parameters were analyzed: R_a (arithmetic average), R_y (maximum), R_z (10-point height), R_q (root mean square), and R_t (maximum peak to valley). Roughness parameters are critical-to-quality characteristics, often used as technical requirements for mechanical products. Achieving a high level of quality for these quality characteristics is of great importance for the functional behavior of a mechanical part [39]. Figure 4 shows the equipment evaluated in this study, which was a Mitutoyo portable roughness checker set to a cut-off length of 0.8 mm.

5 Turning operation capability analysis

Prioritizing the critical-to-quality characteristic is a common strategy in quality improvement projects. Thus, due to its relevance in most manufacturing processes, the

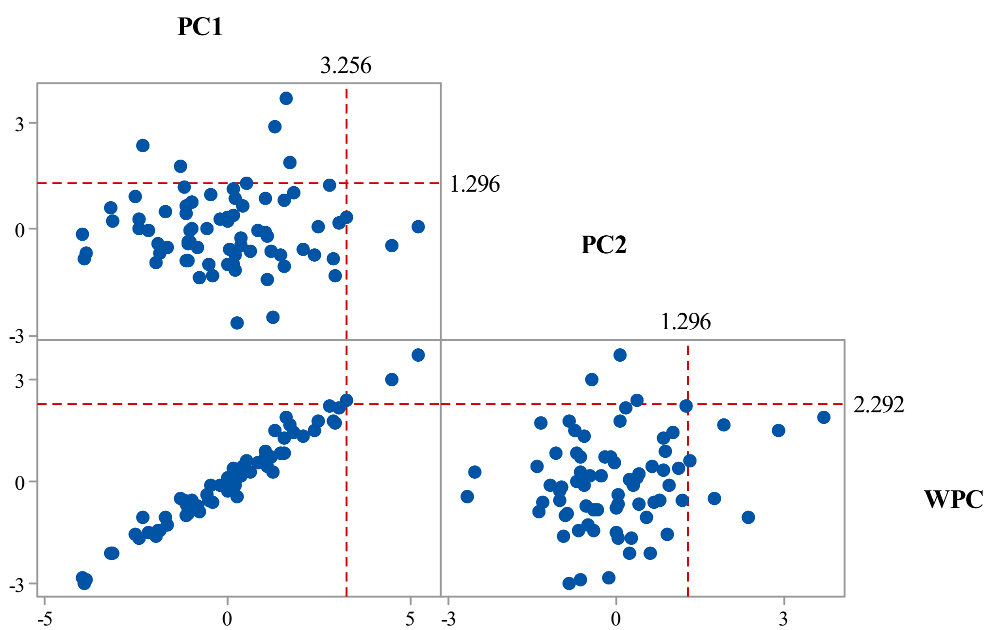


Fig. 7 Scatter plot for principal components PC_1, PC_2, and WPC

Table 4 Normality test, upper specification limit and process capability estimates for principal components

	Normality test	USL_{pci}	$\hat{\sigma}$	$Cp_{PCi}(1,0)$
PC_1	0.284[a] (0.621[b])	3.256	1.876	0.579
PC_2	0.802 (0.032)	− 1.296 (− 1.182[c])	1.066 (0.952[c])	0.405 (0.422[c])
PC_3		− 0.601		
PC_4		− 0.075		
PC_5		− 0.527		
WPC	0.330 (0.510)	2.292		

[a] Anderson-Darling statistic for normality test

[b] p value for normality test

[c] Estimates after Johnson transformation

roughness parameter R_a was selected [40]. The dataset for turning the operation capability analysis is shown in Table appendix table 5 and Figs. 5–6. Minimum roughness is better; thereby, only the upper specification limit USL = 0.8 μm was provided. As shown in Table 1, parameter R_a was well adjusted to a normal distribution and the process capability index $Cp(1,0)$ could be estimated by Eq. (3). By assessing the arithmetic roughness parameter R_a, $C_{pk} = 0.88$ determines that the process is inadequate [16, 21, 41].

Nevertheless, parameter R_a is not enough for modeling the surface roughness of a part. When a non-typical valley or peak is measured, R_a is unable to detect such surface characteristics. A process capability analysis in terms of parameter R_y must be conducted [40], since it is widely known as a measure for checking the deterioration of the part's vertical surface. For parameter R_y, the upper specification limit was $USL =$ 3.29 μm. By using version $Cp(1,0)$ in Eq. (1), C_{pk} could be calculated according to Eq. (3). By assessing the maximum roughness parameter R_y, $C_{pk} = 0.26$ determined that the process is poor [16, 21, 41].

The parameter R_y alone fails to provide sufficient information about the surface of a part. Various forms of surface roughness may have the same R_y value. When assessing R_a and R_y by using univariate statistical techniques, there might be ambiguity with regard to the capabilities of the process. In fact, a more comprehensive surface roughness modeling would take into account not only R_a and R_y but also R_z, R_q, and R_t. Thus, univariate process capability indices for these parameters are summarized in Table 1. All roughness parameters were measured by the same measuring device and correlated to one another (Table 2). Therefore, a multivariate process capability analysis must be conducted to arrive at a final decision with regard to the process' ability of meeting the specifications.

In this multivariate process capability study, R_a and R_y were assessed, along with R_z, R_q, and R_t. By applying the PC method in Fig. 1, principal component analysis was conducted, and the results are given in Table 3. In Step 3.1, the scores of principal components were calculated by using Eq. (6). Because PC_1 and PC_2 account for 93% of the total variation, only this subset of components was included in the PC method. Figure 7 illustrates a scatter plot of the principal component scores used in this study. In Step 4, since the PC_2 vector cannot be adjusted to a normal distribution (Table 4), a Johnson transformation ($PC_2^* = -0.724 + 1.730 \times A\sinh((X + 0.731) \quad /1.514) \quad$) was applied to estimate $Cp_{PCi}(1,0)$. In Step 5, the upper specification limits, in terms of principal components, were calculated by Eq. (8) to estimate $M_1Cp(1,0)$, $M_2Cp(1,0)$, and $M_3Cp(1,0)$. For Step 6, Table 4 summarizes the process capability indices $Cp_{PC_1}(1,0)$ and $Cp_{PC_2}(1,0)$ for each principal component using Eq. (3). Eventually, by using Eqs. (10)–(12), the multivariate capability indices in Step 7 were calculated based only on PC_1 and PC_2. According to $M_1Cp(1,0) = 0.494$,$M_2Cp(1,0) =$ 0.536, and$M_3Cp(1,0) = 0.503$, this multivariate hard turning process was unable to meet the specifications and was deemed as inadequate.

At this point, by focusing on the proposed multivariate index, the weighted scores of principal components in Step 3.1 of Fig. 2 were calculated by using Eq. (14) and can be seen in Fig. 7. It is important to highlight that the proposed method takes into account all principal components for obtaining the WPC vector. Before assessing the capability of the process, in Step 4, the normality assumption on the WPC vector was checked and confirmed by the Anderson–Darling test, as in Table 4. For Step 5, the upper specification limits, in terms of weighted principal component, were calculated with Eq. (16). By using Eq. (18) in Step 6, $M_{wpc}Cp(1,0) = 0.461$ also indicates that the multivariate process is inadequate.

In summary, the proposed method was successful since it was able to provide similar results in relation to methods cited in the literature. Nevertheless, some obvious advantages of the proposed procedure should be highlighted. First, while the methods in the literature have been applied to a few principal components, the WPC method was applied to all PCs and 100% of the original variation was taken into account. Secondly, to estimate the methods cited in the literature, a normality test should be conducted for all PCs in Eqs. (10)–(12), while the WPC method requires only one normality test. Eventually, the process stability studies based on literature methods would require the evaluation of control charts for each principal component before estimating the process capability. In the proposed method, a control chart procedure would be applied only to the WPC vector. Finally, further investigation must be conducted to investigate how the setup of control variables should be changed to improve the overall process capability. Some useful optimization methods such as [1, 7, 35] could be used to address this issue.

6 Conclusion

Hard turning can eliminate the grinding process in machining hardened materials. For this, a capability analysis should be conducted to verify if the product meets the specifications. Since these processes usually present multiple quality characteristics, this study aimed to explore the multivariate analysis of the capability to measure the roughness of the pieces. By evaluating the process capability based on the R_a parameter, the case was classified as inadequate, such that $C_p = 0.88$. However, parameter R_y, which evaluates the dispersion of the roughness profile, classified the process as poor, such that $C_p = 0.26$. Hence, the univariate approach was unable to come up with a final decision regarding the ability of the process to meet specifications.

Literature and the proposed methods for estimating multivariate capability indices were performed. The results based on *PC* and *WPC* methods classified the hard turning process as inadequate. This study indicated that the weighting scores of principal components were also a reasonable approach to estimate the multivariate capability indices. Moreover, the proposed method depended on fewer assumptions when performing multivariate process capability analysis, in comparison to literature methods. As highlighted in the previous section, the power of reducing the problem dimension was greater with regard to conducting the weighted principal component method.

Further investigation needs to be carried out to investigate the effectiveness of the weighted principal component method in scenarios with distinct correlation structures and process capability requirements. Moreover, a multivariate optimization procedure should be applied to improve the multivariate process capability indices.

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Appendix

Table 5 Dataset of the hard turning experiment for process capability assessment

R_a	R_y	R_z	R_q	R_t	R_a	R_y	R_z	R_q	R_t	R_a	R_y	R_z	R_q	R_t
0.70	3.24	2.65	0.80	3.28	0.78	3.66	2.90	0.91	3.76	0.63	2.54	2.23	0.70	2.54
0.66	3.23	2.80	0.76	3.23	0.67	2.94	2.67	0.77	3.49	0.68	3.09	2.60	0.77	3.28
0.69	3.03	2.85	0.83	3.21	0.68	2.95	2.53	0.77	2.95	0.65	3.05	2.52	0.73	3.24
0.64	2.93	2.49	0.75	3.37	0.65	3.18	2.54	0.75	3.31	0.65	3.11	2.71	0.75	3.51
0.66	2.80	2.48	0.75	2.86	0.72	2.90	2.46	0.80	2.96	0.63	2.70	2.54	0.72	3.24
0.61	2.70	2.30	0.69	2.96	0.71	3.09	2.60	0.79	3.09	0.71	2.78	2.53	0.78	2.88
0.67	3.04	2.43	0.76	3.04	0.70	3.01	2.63	0.79	3.31	0.63	2.26	2.21	0.71	2.76
0.69	2.86	2.60	0.79	2.92	0.66	2.85	2.56	0.75	3.15	0.67	2.92	2.57	0.75	2.93
0.64	3.60	2.61	0.76	4.06	0.68	3.36	2.71	0.78	3.47	0.68	3.20	2.51	0.77	3.20
0.67	3.33	2.59	0.76	3.58	0.77	3.21	2.81	0.86	3.64	0.66	4.00	2.59	0.72	4.10
0.67	3.24	2.57	0.77	3.44	0.73	2.90	2.78	0.83	3.15	0.67	3.03	2.63	0.75	3.05
0.71	3.27	2.68	0.81	3.27	0.69	2.87	2.71	0.78	3.16	0.67	2.93	2.52	0.76	2.93
0.62	2.72	2.50	0.72	2.83	0.69	2.88	2.48	0.77	2.88	0.71	3.61	2.91	0.80	3.79
0.66	2.84	2.42	0.74	2.84	0.70	2.89	2.76	0.80	3.27	0.61	2.68	2.42	0.69	2.77
0.78	2.80	2.69	0.85	3.04	0.71	2.77	2.68	0.79	3.19	0.67	2.55	2.43	0.75	2.92
0.77	2.71	2.58	0.83	2.86	0.67	3.17	2.69	0.76	3.17	0.70	2.92	2.70	0.79	3.13
0.74	3.19	2.85	0.85	3.46	0.72	2.94	2.57	0.79	3.06	0.61	2.48	2.30	0.68	2.68
0.61	2.92	2.39	0.70	2.98	0.58	3.21	2.42	0.67	3.21	0.69	2.74	2.52	0.76	2.96
0.72	3.53	2.97	0.85	3.65	0.77	3.82	3.13	0.90	3.82	0.73	3.20	2.87	0.83	3.38
0.69	3.01	2.74	0.79	3.13	0.77	3.24	2.90	0.87	3.36	0.67	2.77	2.54	0.76	3.11
0.67	3.29	2.81	0.81	3.60	0.74	3.10	2.77	0.83	3.24	0.65	2.68	2.53	0.75	2.81
0.71	3.34	2.91	0.84	3.53	0.73	3.15	2.76	0.82	3.28	0.63	3.03	2.58	0.74	3.09
0.69	3.27	2.71	0.81	3.87	0.64	2.80	2.47	0.72	2.87	0.62	3.13	2.60	0.72	3.13
0.71	3.27	2.69	0.80	3.27	0.73	2.97	2.58	0.80	3.06	0.63	2.91	2.64	0.74	3.07
0.66	3.66	2.79	0.78	3.66	0.63	2.84	2.41	0.71	2.87	0.62	3.28	2.46	0.71	3.28
0.73	3.56	2.90	0.85	3.56										

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